(7 pages) **Reg. No. :**.....

Code No. : 20594 E **Sub. Code : SSMA 4 A**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021.

Fourth Semester

Mathematics

Skill Based Subject — TRIGONOMETRY, LAPLACE TRANSFORMS AND FOURIER SERIES

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. 1^0 = _____ radians.

(a) π (b) $\frac{\pi}{180}$

(c) $\frac{\pi}{90}$ (d) 2π

2. $\sinh^{-1} x$ = _____.

(a) $\log_e\left(x+\sqrt{x^2-1}\right)$ (b) $\pm\log_e\left(x+\sqrt{x^2-1}\right)$

(c) $\log_e\left(x+\sqrt{x^2+1}\right)$ (d) $\pm\log_e\left(x+\sqrt{x^2+1}\right)$

3. When θ is expressed in radians, $\sin\theta$ = _____.

(a) $\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}+....$ (b) $1+\theta-\frac{\theta^2}{2}+\frac{\theta^3}{3}-....$

(c) $1+\theta+\frac{\theta^2}{2}+\frac{\theta^3}{3}+....$ (d) $\theta+\frac{\theta^3}{3}+\frac{2\theta^5}{15}+....$

4. $\tanh x$ = _____.

(a) $\tan x$ (b) $\tan(ix)$

(c) $i\tan(ix)$ (d) $-i\tan(ix)$

5. $L\left(e^{2x}\right)$ = _____.

(a) $\frac{1}{s+2}$ (b) $\frac{1}{s-2}$

(c) $\frac{1}{s}$ (d) 1

6. Value of $L^{-1}\left(\frac{1}{s}\right)$ is

(a) 1 (b) 0

(c) x (d) $\frac{1}{x}$

7. Value of $L(\sinh ax)$ is

(a) $\frac{a}{s^2}$ (b) $\frac{a}{(s+a)^2}$

(c) $\frac{a}{s^2-a^2}$ (d) $\frac{a}{s^2+a^2}$

8. Value of $L^{-1}\left(\frac{s}{a^2s^2+b^2}\right)$ is

(a) $a\cos bx$ (b) $\frac{1}{a}\cos bx$

(c) $a^2\cos\left(\frac{bx}{a}\right)$ (d) $\frac{1}{a^2}\cos\left(\frac{bx}{a}\right)$

9. $f(x)$ is an even function of $f(-x)$ = _____.

(a) $f(x)$ (b) $-f(x)$

(c) $f(x^2)$ (d) $-f(x^2)$

10. In the interval $(-\pi, \pi)$, the Fourier Co-efficient $a_n =$ _____.

(a) $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx$ (b) $\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx$

(c) $\frac{1}{\pi}\int_{-\pi}^{\pi} \cos nx dx$ (d) $\frac{1}{2\pi}\int_{-\pi}^{\pi} \cos nx dx$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that

$$\cos n\theta = \cos^n \theta - {}^nC_2 \cos^{n-2}\theta \sin^2\theta + \ldots$$

Or

(b) Prove that

$$2^5 \cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10 .$$

12. (a) Prove that $\frac{1+\tanh x}{1-\tanh x} = \cosh 2x + \sinh 2x$.

Or

(b) If $\cosh u = \sec\theta$, prove that $u = \log_e \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$.

13. (a) Find $L\left(t^2+\cos 2t\cos t+\sin^2 t\right)$.

Or

(b) Find $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right]$.

14. (a) Using Laplace transform, solve $y'+3y=e^{-2x}$, given $y(0)=4$.

Or

(b) Find $L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right]$.

15. (a) Find the sine series for the function $f(x)=k$, $0<x<\pi$.

Or

(b) Find the cosine series for the function $f(x)=\pi-x$ in the interval $(0,\pi)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that

$\cos 8\theta=128\cos^8\theta-256\cos^6\theta+160\cos^4\theta-32\cos^2\theta+1$.

Or

(b) When $n \in \mathbb{Z}^+$,

$$\cos^n \theta = \frac{1}{2^{n-1}}\left[\cos n\theta + nC_1 \cos(n-2)\theta + nC_2 \cos(n-4)\theta + \ldots\right]$$

Prove.

17. (a) Prove that

$$Log\left(\frac{1}{1-e^{i\theta}}\right) = \log\left(\frac{coec\left(\frac{\theta}{2}\right)}{2}\right) + i\left(2n\pi + \frac{\pi}{2} - \frac{\theta}{2}\right).$$

Or

(b) Find the sum to infinity the series

$$1 + \cos\theta\cos\theta + \cos^2\theta\cos 2\theta + \cos^3\theta\cos 3\theta + \ldots\infty .$$

18. (a) (i) Prove that

$$L(f''(x)) = s^2 L(f(x)) - sf(0) - f'(0).$$

(ii) Find $L^{-1}\left(\frac{s}{(s+2)^2}\right)$.

Or

(b) Find the value of $L^{-1}\left(\frac{1+2s}{(s+2)^2(s-1)^2}\right)$.

19. (a) Solve by using Laplace transform :

$y''+4y'+13y = 2e^{-x}$, given $y(0)=0$, $y'(0)=-1$.

Or

(b) Solve by using Laplace transform :

$\frac{dx}{dt}+y=\sin t$, $\frac{dy}{dt}+x=\cos t$ given $x(0)=2$, $y(0)=0$.

20. (a) Show that $x^2=\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}\left[\frac{(-1)^n\cos nx}{n^2}\right]$ in $-\pi<x<\pi$. Deduce that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-....=\frac{\pi^2}{12}$.

Or

(b) Find the Fourier Expansion $f(x)=x$ in the interval $(-\pi, \pi)$.

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